**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. **0.2676**
4. 0.5
5. 0.6987

X= 60,mean = 55,stddev = 8. Calculate Z Score

P(x>60)= 1-stats.norm.cdf(60,55,8)

= 0.26

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

False

Probability of employees age> 44 = 15% = 15%\*400=60

Probability of employee age between 38 and 44 = 34% =34%\*400=136

More employees are in the range of 38 and 44 age.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

True

Probability of employees age <30 = 91%;

No. of Employees = 91%\*400 = 36

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

2X1=N(2μ,2σ2)

X1+X2=N(2μ,2σ2)

2X1-(X1+X2)=

= As we know that if X ∼ N(µ1, σ1^2 ), and Y ∼ N(µ2, σ2^2 ) are two independent random variables then X + Y ∼ N(µ1 + µ2, σ1^2 + σ2^2 ) , and X − Y ∼ N(µ1 − µ2, σ1^2 + σ2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore in the question

2X1~ N(2 u,4 σ^2) and

**X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 u, 2σ^2 )**

**2X1-(X1+X2) = N( 4µ,6 σ^2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A and b values are min value and max value of 99% confidence interval.

We solve this using the below code

stats.norm.interval(0.99,100,20)

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company

**# Profit1 ~ N(5, 3^2)**

**#Profit2 ~ N(7, 4^2)**

**# So Overall Profit ~ N(12,25)**

**mean= 12**

**sd=5**

**mean\_rs= 12\*45**

**sd\_rs= 5\*45**

**print(mean\_rs)**

**print(sd\_rs)**

**ANS : 540**

**225**

**\* mean profit from two divisions is 540 million rupees**

**\* Standard deviation of mean profit is 225 million rupees**

**# getting rupee range such that it contains 95% probability for annual profit**

**stats.norm.interval(0.95,540,225)**

**ANS:(99.00810347848784)(980.9918965215122)** **\* rupee range in between 99 million to 981 million.**

B . Specify the 5th percentile of profit (in Rupees) for the company

**# 5th percentile= area of 0.05 in normal distribution curve**

**# calculating z score for corresponding probability**

**stats.norm.ppf(0.05,loc=0,scale=1)**

**-1.6448536269514729**

z**= -1.65**

**fifth\_percentile=z\*sd\_rs+mean\_rs**

**print("5th % of profit in million rupees:",fifth\_percentile)**

**5th % of profit in million rupees:168.75**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**P(Profit1<0)**

**Stats.norm.cdf(0,5,3)**

**0.048**

**4.8%(This division is likely to make more loss)**

**P(Profit2<0)**

**Stats.norm.cdf(0,7,4)**

**0.04**

**4%**

**\* Probability of loss is greater for division 1**